The West African Examinations Council

*Senior High School Certificate Examination*

# MATHEMATICS

May 2012 Exam Solutions

Prepared by Peace Corps, Liberia

301 S.H.S.C.E
MAY 2012
MATHEMATICS
Objective and Essay Tests
3hours

1&2

Name:__________________________

# SOLUTIONS

## THE WEST AFRICAN EXAMINATIONS COUNCIL

Senior High School Certificate Examination

May 2012 MATHEMATICS 1 ½ hours

**PAPER 1**
OBJECTIVE TEST

*Exam questions are listed on the left and solutions are given to the right. Please note these solutions are intended as a guide and a study tool. They are not designed to teach you how to solve each problem. Independent research and study will still be necessary if a concept is unfamiliar.*

1. Simplify

$$\frac{(0.000006)(0.03 \times 10^{16})}{(30{,}000)(600 \times 10^{-11})}$$

A. $10^{13}$
B. $10^{32}$
C. $10^{-32}$
D. $10^{-24}$

*This question requires you to use the rules of exponents and your knowledge of* ***scientific notation****.*

*Remember that in scientific notation you write very large numbers and very small numbers as an integer multiplied by a power of 10. This makes the operations easier, espeically without a calculator. The power on the 10 represents the number of places you move the decimal. When it moves to the left the power is negative. When it moves to the right the power is positive.*

*So you can rewrite this problem to look like:*

$$\frac{(6 \times 10^{-6})(3 \times 10^{14})}{(3 \times 10^{4})(6 \times 10^{-9})}$$ *Change the number in each bracket to scientific notation*

$$\frac{(\cancel{6} \times 10^{-6})(\cancel{3} \times 10^{14})}{(\cancel{3} \times 10^{4})(\cancel{6} \times 10^{-9})}$$ *Cancel 3 and 6 because they appear in both the numerator and denominator*

$$\frac{(10^{-6})(10^{14})}{(10^{4})(10^{-9})}$$ *Only powers of 10 are left*

$$\frac{10^{-6+14}}{10^{4+(-9)}}$$ *Simplify using the Product Rule*

$$\frac{10^{8}}{10^{-5}}$$

$$10^{8-(-5)}$$ *Simplify using the Quotient Rule*

$$10^{8+5}$$ *Minus a negative is plus a positive*

$$10^{13}$$

2. Sonsiama had some money saved. Starting the first week of June, he saved \$6.00 a week for a new bicycle. At the end of 16 weeks he had a total of \$141.00 saved. How much did he save before June?

   A. \$8.44
   B. \$8.81
   C. \$45.00
   D. \$96.00

Step #1 *UNDERSTAND*

*What do we <u>want</u> to know?*

- *How much did he save before June?*

*What <u>do</u> we know?*

- *He saved \$6 each week starting in June*
- *After 16 weeks he had saved a total of \$141.00*

Step #2 *PLAN*

Assign Variables

*w = number of weeks he has saved since June*
*p = amount of previous savings*

Write an equation to express the problem

$$6w + p = total\ savings$$

Step #3 *SOLVE AND CHECK*

*Substitute the information from the reading and solve:*

$$\begin{aligned} 6(16) + p &= 141 \\ 96 + p &= 141 \\ p &= 141 - 96 \\ p &= 45 \end{aligned}$$

3. The equation $ax = a + 2x$ has a solution for all values of *a* **except**

   A. $a = 2$
   B. $a = 0$
   C. $a = -1$
   D. $a = -2$

*SHORT CUT! This problem can also be solved by substituting each answer choice into the equation and checking if it is a solution.*

Step #1 *Solve the equation for x:*

$$ax = a + 2x$$

$$ax - 2x = a \qquad \textit{Carry the } 2x \textit{ across}$$

$$x(a-2) = a \qquad \textit{Factor out } x \textit{ on the left}$$

$$x = \frac{a}{(a-2)} \qquad \textit{Divide by } (a-2)$$

Step #2 *The equation will be undefined when the denominator is zero because you cannot cut something into zero pieces. Set it equal to zero and solve:*

$$\begin{aligned} a - 2 &= 0 \\ +2 \quad &+ 2 \\ \hline a &= 2 \end{aligned}$$

4. In a class of 36 students, 30 students like rice, 12 students like cassava and 1 student likes neither cassava nor rice. How many students like both cassava and rice?

   A. 1
   B. 7
   C. 12
   D. 13

Step #1 *UNDERSTAND*

What do we <u>want</u> to know?

- *How many students like both cassava and rice?*

What do we know?

- *36 students are in the class*
- *30 students like rice*
- *12 students like cassava*
- *1 student likes neither*

Step #2 *PLAN*

Assign Variables:

$r = number\ who\ like\ rice\ only$
$c = number\ who\ like\ cassava\ only$
$n = number\ who\ like\ neither\ rice\ nor\ cassava$
$b = number\ who\ like\ both$
$t = total\ students\ in\ the\ class$

Write an equation to describe the problem:

$$(r - b) + (c - b) + n + b = t$$

*Students who like <u>both</u> are counted in the 'both' group and also in the 'rice' and 'cassava' groups, so they are counted twice. Subtract b from the number who like rice and the number who like cassava.*

*SHORT CUT!*
*Once you get to*
$43 - b = 36$
*substitute the*
*answer choices*
*for b to find b=7.*

Step #3 *SOLVE AND CHECK*

*Substitute the information given in the problem and solve for b = both*

$$\begin{aligned}(30 - b) + (12 - b) + 1 + b &= 36\\ 30 - b + 12 - b + 1 + b &= 36\\ 43 - b &= 36\\ -b &= 36 - 43\\ -b &= -7\\ b &= 7\end{aligned}$$

*Good mathematicians always check their work. Substitute* $b = 7$ *into your equation and check that it gives you a true statement.*

5. If $y = 3x + 1$, $z = 3x - 4$, and $3x + y + z = 96$, find the numerical value of $x$.

   A. 11
   B. 12
   C. 13
   D. 14

Step #1 *You cannot solve when there is more than one variable you don't know (more than one unknown).* Since you already know the value of $y$ and $z$ in terms of $x$, you can substitute the value for each into the 3rd equation and solve for $x$:

$$\begin{aligned}3x + y + z &= 96\\ 3x + (3x + 1) + (3x - 4) &= 96\end{aligned}$$

Step #2 *Collect like terms and solve for x:*

$$\begin{aligned}9x - 3 &= 96\\ 9x &= 99\\ x &= \frac{99}{9}\\ x &= 11\end{aligned}$$

6. A car which cost \$8,998 when brand new depreciates by about 13% in the first 6 months. Find its value after 6 months to the **nearest** \$100.

   A. \$6,900
   B. \$7,700
   C. \$7,800
   D. \$8,800

*This problem asks you to calculate the* ***percent decrease*** *in the value of the car.*

$$value\ after\ 6\ months = (starting\ value) - (loss\ in\ value)$$
$$= (cost\ price) - \left(\frac{13}{100} \times cost\ price\right)$$

*Since you are asked to round the answer to the nearest \$100, round the cost price of \$8,998 up to \$9,000 to make your calculations easier.*

$$value\ after\ 6\ months = (9{,}000) - \left(\frac{13}{100} \times 9{,}000\right)$$
$$= (9{,}000) - \left(\frac{117{,}000}{100}\right)$$
$$= (9{,}000) - \left(\frac{117{,}0\cancel{00}}{\cancel{100}}\right)$$
$$= (9{,}000) - (1{,}170)$$
$$= 7{,}830$$
$$= 7{,}800\ rounded\ to\ the\ nearest\ 100$$

7. An athlete runs to the top of a hill and back down. His average speed uphill is $6\frac{km}{hr}$ and his average speed downhill is $12\frac{km}{hr}$. What is the average speed for the whole journey?

   A. 6 km/hr
   B. 7 km/hr
   C. 8 km/hr
   D. 9 km/hr

*This problem asks you to calculate a* ***weighted average*** *since the athlete doesn't spend an equal amount of time on each side of the hill. Let* $d = distance$, $t_1 = time\ uphill$ *and* $t_2 = time\ downhill$. *Then using the formula for speed:*

$$speed = \frac{distance}{time}$$

| *UPHILL* | *DOWNHILL* |
|---|---|
| $6\frac{km}{hr} = \frac{d}{t_1}$ | $12\frac{km}{hr} = \frac{d}{t_2}$ |
| $t_1 = \frac{d}{6}hrs$ | $t_2 = \frac{d}{12}hrs$ |

*Then, since* $2 \times 6 = 12$ *you can factor the equation for time spent downhill and substitute to write:*

$$t_2 = \frac{d}{12}hrs = \frac{d}{2 \cdot 6}hrs = \frac{1}{2}(t_1)$$

*If you assume the distance is* $6km$, *then the athlete spends* $1hr$ *going up and ½ hour going down. So you can see he spends two times as much time going up as he does going down. Write the equation:*

$$average\ speed = \frac{2(speed\ uphill) + 1(speed\ downhill)}{2+1}$$
$$= \frac{2\left(6\frac{km}{hr}\right) + 1\left(12\frac{km}{hr}\right)}{3}$$
$$= 8\frac{km}{hr}$$

8. Simplify $4\sqrt{50} + 10\sqrt{200} - 16\sqrt{288}$

A. $-192\sqrt{2}$
B. $-72\sqrt{2}$
C. $120\sqrt{2}$
D. $312\sqrt{2}$

*Rewrite each radicand (the part under the radical sign) as the product of* ***perfect squares****. Remember a perfect square is a number you get by squaring another number. For example in* $2^2 = 4, 3^2 = 9$ *and* $4^2 = 16$ *the numbers 4, 9, and 16 are perfect squares because you can get them by squaring another number.*

$$4\sqrt{50} + 10\sqrt{200} - 16\sqrt{288}$$

$$4\sqrt{2 \cdot 25} + 10\sqrt{2 \cdot 100} - 16\sqrt{2 \cdot 144}$$ *Write each radicand as the product of perfect squares*

$$4\sqrt{25}\sqrt{2} + 10\sqrt{100}\sqrt{2} - 16\sqrt{144}\sqrt{2}$$ *Take the radical of each factor separately*

$$4\sqrt{5^2}\sqrt{2} + 10\sqrt{10^2}\sqrt{2} - 16\sqrt{12^2}\sqrt{2}$$ *Rewrite 25, 100, and 144 as a number squared*

$$4(5)\sqrt{2} + 10(10)\sqrt{2} - 16(12)\sqrt{2}$$ *A square root cancels a square*

$$20\sqrt{2} + 100\sqrt{2} - 192\sqrt{2}$$ *Multiply*

$$(20 + 100 - 192)\sqrt{2}$$ *Factor out* $\sqrt{2}$

$$-72\sqrt{2}$$ *Simplify*

9. Nunneh sold an article to Bondo at a profit of 20%. Bondo sold it to Fania at a loss of 20% of what it cost him. What is the ratio *final price : original price?*

A. 25 : 36
B. 24 : 25
C. 5 : 7
D. 4 : 5

Step #1 *Remember that a ratio is just another way to write a division problem so:*

$$final\ price : original\ price = \frac{final\ price}{original\ price}$$

Step #2 *Write an equation to express a 20% profit and solve it for the original price*

$$(original\ price)\left(1 + \frac{20}{100}\right) = (selling\ price)$$

$$(original\ price)(1.2) = (selling\ price)$$

$$(original\ price) = \frac{(selling\ price)}{1.2}$$

Step #3 *Write an equation to express the final price as a 20% loss on the selling price from step 2*

$$(final\ price) = (selling\ price)\left(1 - \frac{20}{100}\right)$$

$$= (selling\ price)(0.8)$$

Please Turn Over

Step #4 *Substitute the equations from step 2 and step 3 into the step 1 equation:*

$$\frac{final\ price}{original\ price} = \frac{(selling\ price)(0.8)}{\left(\frac{selling\ price}{1.2}\right)}$$

$$= (\cancel{selling\ price})(0.8)\left(\frac{1.2}{\cancel{selling\ price}}\right)$$

$$= (0.8)(1.2)$$

$$= \left(\frac{8}{10}\right)\left(\frac{12}{10}\right)$$

$$= \frac{96}{100}$$

$$= \frac{24}{25}$$

$$= 24 : 25$$

10. Which of the following is equal to $\frac{5}{\sqrt{5}-\sqrt{2}}$?

   A. $\frac{5}{7}(\sqrt{5}+\sqrt{2})$

   B. $\frac{5}{7}(\sqrt{5}-\sqrt{2})$

   C. $\frac{5}{3}(\sqrt{5}-\sqrt{2})$

   D. $\frac{5}{3}(\sqrt{5}+\sqrt{2})$

*It is improper to have a radical in the denominator of a fraction. Multiply by the* ***conjugate*** *of the denominator to fix this. The conjugate is formed by changing the sign between the terms $\sqrt{5}$ and $\sqrt{2}$. Then since anything divided by itself is equal to one you must also multiply it by the numerator so the problem is not changed. Then by simplifying:*

$$\frac{5}{\sqrt{5}-\sqrt{2}} = \frac{5}{\sqrt{5}-\sqrt{2}}\left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}\right)$$

$$= \frac{5(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}$$

$$= \frac{5(\sqrt{5}+\sqrt{2})}{(\sqrt{5}\sqrt{5}) + (\sqrt{5}\sqrt{2}) + (-\sqrt{2}\sqrt{5}) + (-\sqrt{2}\sqrt{2})}$$

$$= \frac{5(\sqrt{5}+\sqrt{2})}{5 + \sqrt{5}\sqrt{2} - \sqrt{5}\sqrt{2} - 2}$$

$$= \frac{5(\sqrt{5}+\sqrt{2})}{5-2}$$

$$= \frac{5(\sqrt{5}+\sqrt{2})}{3}$$

$$= \frac{5}{3}(\sqrt{5}+\sqrt{2})$$

11. Evaluate $36^{log_6 5}$

A. 16
B. 25
C. 36
D. 49

*Rewrite the expression and use the properties of logarithms to simplify it*

$36^{log_6 5}$

$6^{2(log_6 5)}$ *Rewrite 36 as a power of 6*

$6^{(log_6 5^2)}$ *Apply the Power Property of logarithms*

$\cancel{6}^{(\cancel{log_6} 5^2)}$ *Base 6 and $log_6$ cancel each other*

$5^2$

25

12. Find the coefficient of the term containing $x^7$ in the expression $(x - y)^{10}$.

A. −240
B. −120
C. 120
D. 240

*This problem asks us to perform* ***binomial expansion****. You can do this using either Pascal's Triangle or the Binomial Theorem. Both methods are presented, but may require further study for good understanding.*

Method #1 *Start by creating a Pascal's Triangle with 11 rows:*

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

*The last row will give you the coefficients because the second number is 10 and we are asked to expand a binomial to the 10th power. So write:*

$$(x-y)^{10} = x^{10}(-y)^0 + 10x^9(-y)^1 + 45x^8(-y)^2 + 120x^7(-y)^3 + 210x^6(-y)^4 + 252x^5(-y)^5 + 210x^4(-y)^6 + 120x^3(-y)^7 + 45x^2(-y)^8 + 10x^1(-y)^9 + x^0(-y)^{10}$$

*The entire expansion is shown here but on the exam you can stop when you reach the term $120x^7(-y)^3$ because it is the only one you are asked about. This will save valuable time!*

*The question asks about the coefficient of the term containing $x^7$ so look only at:*

$120x^7(-y)^3$

$120x^7(-1)^3(y)^3$

$-120x^7y^3$

*So the coefficient of the term containing $x^7$ is $-120$*

Method #2

*The **Binomial Theorem** can be used to find a particular term of a binomial expansion without writing out the entire expression. The formula for finding the $k^{th}$ term of an expansion is:*

$$(x+y)^n = \left(\frac{n!}{k!\,(n-k)!}\right) x^{n-k} y^k$$

*When counting the terms you start at zero instead of one so take note that $k = (term\#) - 1$. In this problem you need the fourth term so $k = 4 - 1 = 3$.*

*Substituting the information given in the reading:*

$$(x-y)^{10} = \frac{10!}{3!\,(10-3)!}\mathrm{x}^{10-3}(-\mathrm{y})^3$$

$$= \frac{10!}{3!\,7!}\mathrm{x}^7(-\mathrm{y})^3$$

$$= \frac{10\cdot 9\cdot 8\cdot \cancel{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}{(3\cdot 2\cdot 1)(\cancel{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1})}\mathrm{x}^7(-\mathrm{y})^3$$

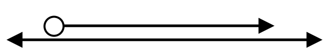
$$= \frac{720}{6}\mathrm{x}^7(-\mathrm{y})^3$$

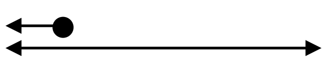
$$= 120\mathrm{x}^7(-1)^3(\mathrm{y})^3$$

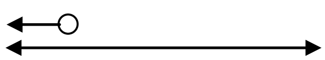
$$= -120x^7y^3$$

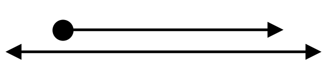
*So the coefficient is $-120$*

13. Find and graph the solution set: $2(2x-8) - 8x \le 0$.

A.



B.



C.



D.



*Remember, you treat an inequality sign the same way you treat an equal sign so this is like solving an equation for x.*

$2(2x-8) - 8x \le 0$ *Distribute the 2 to the brackets*

$4x - 16 - 8x \le 0$ *Combine like terms*

$-16 - 4x \le 0$ *Carry the 4 across*

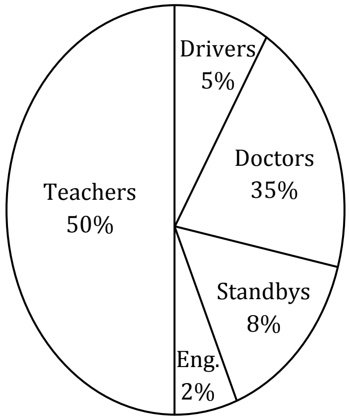
$-16 \le 4x$ *Divide by 4*

$-4 \le x$

*When graphing an inequality use an open circle ○ to indicate that the end point is <u>not</u> included in the solution (< or >) and a darkened ● circle is used if an end point <u>is</u>* included in the solution (≤ or ≥).

14. The pie chart below shows how a company intends to employ 500 persons in 2012 as drivers, engineers, doctors, teachers, and standbys. How many persons will be on standby?

A. 250
B. 175
C. 40
D. 25



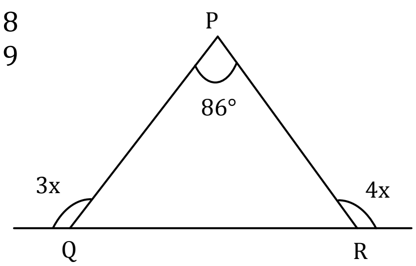

*You can see from the pie chart that 8% of the employees are on standby. This problem is a complicated way of asking us to find 8% of 500.*

$$(\%on\ standby)(total\ employees) = (\#\ on\ standby)$$

$$\left(\frac{8}{100}\right)(500) = 40$$

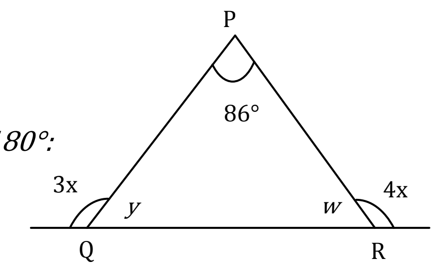
15. Calculate the value of **x** in $\Delta PQR$ below.

A. 13.4
B. 14.9
C. 38
D. 39




Method #1 *Straight angles are always 180° and interior angles add to 180° so you can write:*

$$3x + y = 180°\ so\ y = 180° - 3x$$
$$4x + w = 180°\ so\ w = 180° - 4x$$



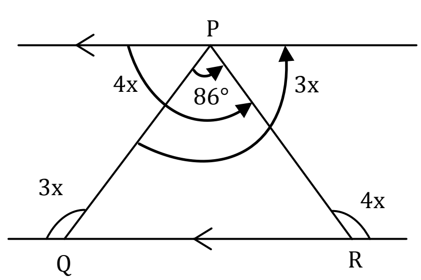

*The angles of a triangle always add to 180°:*

$$y + w + 86° = 180°$$

*Then by substituting and solving:*

$$\begin{aligned}(180° - 3x) + (180° - 4x) + 86° &= 180°\\ 180° + 180° + 86° - 3x - 4x &= 180°\\ 446° - 7x &= 180°\\ -7x &= 180° - 446°\\ -7x &= -266°\\ x &= 38°\end{aligned}$$

Method #2 *If you draw a line through P that is parallel to $\overleftrightarrow{QR}$ we know 3x and 4x reflect up and are the measures of their **alternate interior angles**. Then since a straight line measures 180° we know:*




$$4x + 3x - 86 = 180$$
$$7x - 86 = 180$$
$$7x = 266$$
$$x = \frac{266}{7}$$
$$x = 38°$$

*The 86° must be subtracted because it is included in the 4x and the 3x so it is counted twice.*

16. If $i^2 = -1$ and $a$ is a positive integer, which of the following must be equal to $i^a$?

A. $i^{a+1}$
B. $i^{a+2}$
C. $i^{a+4}$
D. $i^{2a}$

*To solve this problem we need to find the pattern in the powers and see when the solutions repeat*

| | |
|---|---|
| $i^2 = -1$ | $i^a$ |
| $i^3 = -i$ | $i^{a+1}$ |
| $i^4 = 1$ | $i^{a+2}$ |
| $i^5 = i$ | $i^{a+3}$ |
| $i^6 = -1$ | $i^{a+4}$ |

$i^2{=}i^6$ so it follows that $i^a{=}i^{a+4}$

17. If $4^x(2^2) = \frac{1}{16}$, what is the value of x?

A. $-4$
B. $-3$
C. $1$
D. $2$

*This problem tests your knowledge of the rules of exponents.*

$$4^x(2^2) = \frac{1}{16}$$

$$(2^2)^x(2^2) = \frac{1}{2^4}$$ *Rewrite 4 and 16 as powers of 2*

$$2^{2x}2^2 = \frac{1}{2^4}$$ *Power Rule*

$$2^{2x}2^2 = 2^{-4}$$ *Negative Exponent Rule*

$$2^{2x+2} = 2^{-4}$$ *Product Rule*

$$2x + 2 = -4$$ *The base on both sides is 2 so set the exponents equal to each other*

$$2x = -6$$ *Carry 2 across*

$$x = -3$$

18. If $y = mx + 3$ and $y = nx - 1$ are equations of perpendicular lines, then $mn =$

A. $-2$
B. $-1$
C. $1$
D. $2$

*Each equation is written in* ***slope-intercept form*** $y = mx + b$ *where the coefficient* $m$ *gives the slope of the line. You can see the slopes of the two equations given are* $m$ *and* $n$*.*

$$y = mx + 3 \qquad y = nx - 1$$

*By definition the slopes of two perpendicular lines are opposite and reciprocal. This means that if a line has slope* $m$ *then the line perpendicular to this first line will have slope* $-\frac{1}{m}$*. So in this problem:*

$$m = -\frac{1}{n}$$
$$mn = \left(-\frac{1}{n}\right)n$$
$$= -\frac{n}{n}$$
$$= -1$$

19. Only 86% of the guests at a wedding reception were invited. How many people attended the reception if 147 of those present were uninvited?

A. 1005
B. 1050
C. 1264
D. 6450

*First collect your data:*

$$\# \, Uninvited = 147$$
$$Percent \ invited = 86$$
$$Percent \ uninvited = 100 - 86 = 14$$
$$\# \, Invited = unknown = x$$

*Since a percent is just a ratio of a part to the total, you can use this information to find the unknown (total attendees).*

$$\% \, univited = 14\% = \frac{14}{100}$$

*Write a proportion to relate the quantities. The total number of guests is the unknown, so you replace it with* $x$ *and solve.*

$$\frac{\# \, uninvited}{\# \, total \ guests} = \% \, uninvited$$

$$\frac{147}{x} = \frac{14}{100}$$

$$14x = 14{,}700$$

$$x = \frac{14{,}700}{14}$$

$$x = 1{,}050$$

20. Solve for $x$: $\frac{3}{x-7} + 5 = \frac{8}{x-7}$

A. 4
B. 5
C. 6
D. 8

*Notice that the fractions both have* $(x-7)$ *as the denominator so they can be combined if one is carried across. Then the problem becomes simple algebra.*

$$\frac{3}{x-7} + 5 = \frac{8}{x-7}$$

$$5 = \frac{8}{x-7} - \frac{3}{x-7}$$

$$5 = \frac{5}{x-7}$$

$$5(x-7) = 5$$

$$x - 7 = \frac{5}{5}$$

$$x - 7 = 1$$

$$x = 8$$

21. Bakeri is 2 inches taller than Gayei and Tambo is 3 inches shorter than Bakeri. If Tambo is 4ft 9inches tall, how tall is Gayei?

A. 5ft 1 in.
B. 5ft
C. 4ft 11in.
D. 4ft 10in.


> *Good mathematicians always check their work. Substitute the values for b,g, and t into each equation and make sure they give true statements.*

Step #1 *UNDERSTAND*

*What do you know?*

- *Bakeri is 2 inches taller than Gayei*
- *Tambo is 3 inches shorter than Bakeri*
- *Tambo is 4ft 9 inches tall*

*What do you want to know?*

- *How tall is Gayei?*

Step #2 *PLAN*

ASSIGN VARIABLES

$b =$ Bakeri's height in inches
$g =$ Gayei's height in inches
$t =$ Tambo's height in inches

WRITE EQUATIONS

*Use your variables to turn each of your 'What do you know?' sentences into an equation.*

$b = g + 2$ Equation #1
$t = b - 3$ Equation #2
$t = 4(12) + 9 = 57$ Equation #3

Step #3 *SOLVE AND CHECK*

*Substitute the value for* $t$ *into the 2nd equation and solve. Then substitute the value for* $b$ *into the 1st equation and solve.*

$$t = b - 3 \qquad b = g + 2$$
$$57 = b - 3 \qquad 60 = g + 2$$
$$b = 60\ inches \qquad g = 58\ inches$$

$$g = 58\ in \times \frac{1\ ft}{12\ in} = 4ft\ 10in$$

22. Which Of the following is **not** in the solution set of $x - 4y \leq 4$?

A. (0,3)
B. (7,1)
C. $(-4, -1)$
D. $(3, -2)$

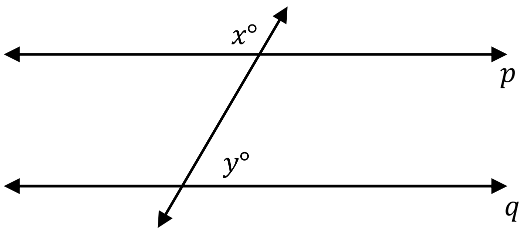
*The fastest way to solve this problem is to substitute each answer choice into the inequality and see which one produces a **false** statement. Remember an ordered pair is always written (x,y).*

| Answer Choice A (0,3) | Answer Choice B (7,1) |
|---|---|
| $0 - 4(3) \leq 4$<br>$0 - 12 \leq 4$<br>$-12 \leq 4$<br>*TRUE* | $7 - 4(1) \leq 4$<br>$7 - 4 \leq 4$<br>$3 \leq 4$<br>*TRUE* |
| **Answer Choice C $(-4, -1)$** | **Answer Choice D $(3, -2)$** |
| $-4 - 4(-1) \leq 4$<br>$-4 + 1 \leq 4$<br>$-3 \leq 4$<br>*TRUE* | $3 - 4(-2) \leq 4$<br>$3 + 8 \leq 4$<br>$11 \leq 4$<br>*FALSE* |

*So* $(3, -2)$ *is **not** in the solution set and D is the answer*

23. In the figure below $p \parallel q$ and $110 < x < 150$. Which is **true** of y?

A. $30 < y < 70$
B. $40 < y < 70$
C. $30 < y < 40$
D. $0 < y < 40$




*Since $p$ and $q$ are parallel lines cut by a transversal, then $x$ and $y$ are supplementary angles such that:*

$$x + y = 180$$
$$y = 180 - x$$

*Then write a new inequality for $y$ in terms of $x$:*

$$(180 - x_1) < y < (180 - x_2)$$
$$(180 - 150) < y < (180 - 110)$$
$$30 < y < 70$$

24. If $f$ and $g$ are inverse functions and $f(x) = 2x - 6$, then

A. $g(x) = \frac{1}{2}x - 3$

B. $g(x) = \frac{1}{2}x + 3$

C. $g(x) = 2x - 3$

D. $g(x) = 2x + 3$

*Rewrite $f(x) = 2x - 6$ as $y = 2x - 6$. Recall that the inverse of a function means that the inputs ($x$) and outputs ($y$) are switched. Then you can write:*

$y = 2x - 6$ *function*

$x = 2y - 6$ *inverse*

*Now solve the invserse for $y$:*

$$x = 2y - 6$$
$$x + 6 = 2y$$
$$\frac{(x+6)}{2} = y$$
$$\frac{1}{2}x + 3 = y$$
$$\frac{1}{2}x + 3 = g(x)$$

*You can check this answer by substituting $g(x)$ into $f(x)$ and $f(x)$ into $g(x)$. If $f$ and $g$ are inverses both problems will solve to $x$. If they result in any other answer they are not inverses.*

$$f(g(x)) = f\left(\frac{1}{2}x + 3\right)$$
$$= 2\left(\frac{1}{2}x + 3\right) - 6$$
$$= \frac{2}{2}x + 2(3) - 6$$
$$= x + 6 - 6$$
$$= x$$

$$g(f(x)) = g(2x - 6)$$
$$= \frac{1}{2}(2x - 6) + 3$$
$$= \frac{2}{2}x - \frac{6}{2} + 3$$
$$= x - 3 + 3$$
$$= x$$

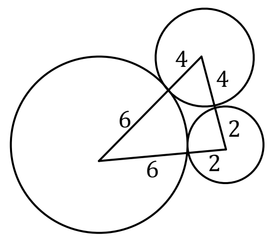
*So the solution checks and the answer is (B).*

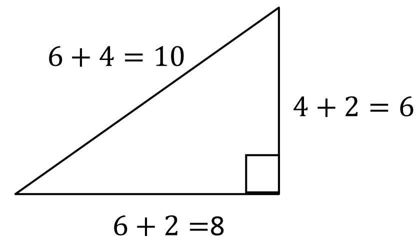
25. Three circles of radii 2, 4, and 6 are tangent to each other externally. Find the area of the triangle formed by connecting their centers.

A. 4
B. 6
C. 12
D. 24

*Draw a diagram that matches the description of the figure given in the reading. Three circles are* ***tangent*** *to each other. That means they must touch at one, and exactly one, point on their circumference.*

*Draw three circles that are touching at one and only one point on their circumference. Then connect the centers with 3 line segments. Label the distance of each side of the triangle that is formed. Note: each side of the triangle is formed by the radii of two circles.*




*The sides of this triangle are in a special relationship called a* ***Pythagorean Triple****. The side lengths 6, 8, and 10 are a solution to the Pythagorean Theorem* $a^2 + b^2 = c^2$. *That means that this is a right triangle with base 8 and height 6.*

Now you can use the formula for the area of a triangle to write:

$$A = \frac{1}{2}bh = \frac{1}{2}(8)(6)$$

$$= (4)(6)$$

$$= 24$$

26. If $3x = 4y - 15$, then $12y - 9x =$

A. 60
B. 45
C. $-45$
D. $-6$

*To solve this problem, you will need to use algebra to rearrange the equation (so that the terms with* $x$ *and* $y$ *are together) and then see if there is a way you can change the first equation into the second, discovering the solution.*

$$\begin{array}{rl}
3x = 4y - 15 & \textit{Carry the 4y across} \\
3x - 4y = -15 & \textit{Multiply both sides by} -3 \\
-3(3x - 4y) = -3(-15) & \textit{Distribute and multiply} \\
-9x + 12y = 45 & \textit{Rearrange the terms} \\
12y - 9x = 45 &
\end{array}$$

27. When using Cramer's Rule to solve the system

$$\begin{cases} 3x - 7y = 11 \\ 5x + 2y = -8 \end{cases}$$

what determinant would be used as denominator of both $x$ and $y$?

A. $\begin{vmatrix} 3 & 11 \\ 5 & -8 \end{vmatrix}$

B. $\begin{vmatrix} 11 & -7 \\ -8 & 2 \end{vmatrix}$

C. $\begin{vmatrix} 3 & -7 \\ 5 & 2 \end{vmatrix}$

D. $\begin{vmatrix} 5 & 3 \\ 2 & -7 \end{vmatrix}$

> *The more common square $[square]$ brackets indicate a matrix and $|straight|$ brackets indicate the determinant of a matrix.*

*A thorough knowledge of Cramer's rule is not necessary to solve this problem so you are encouraged to do self study if you want to know more. All you need to know is that a system of equations can be broken apart and arranged as a product of matrices. It looks like this:*

$$\underbrace{\begin{bmatrix} 3 & -7 \\ 5 & 2 \end{bmatrix}}_{coefficients} \cdot \overbrace{\begin{bmatrix} x \\ y \end{bmatrix}}^{variables} = \underbrace{\begin{bmatrix} 11 \\ -8 \end{bmatrix}}_{solutions}$$

*Cramer's Rule allows us to solve for $x$ and $y$ individually using **determinants**.*

$$x = \frac{D_x}{D} \quad and \quad y = \frac{D_y}{D}$$

*Where $D_x$ is the determinant of a special $x$ matrix and $D_y$ is the determinant of a special $y$ matrix. We only care about the foundation matrix D, however, and that is simple! D is equal the coefficent matrix above so:*

$$D = \begin{vmatrix} 3 & -7 \\ 5 & 2 \end{vmatrix}$$

28. If $\frac{a}{b} = \frac{5}{6}$, find the value of $\frac{b}{2a}$.

A. 5/12
B. 3/5
C. 5/3
D. 12/5

*Given that $\frac{a}{b} = \frac{5}{6}$ it follows that $a = 5$ and $b = 6$. To solve, simply substitute these values for $a$ and $b$ into the fraction $\frac{b}{2a}$ and simplify.*

$$\frac{b}{2a} = \frac{6}{2(5)}$$
$$= \frac{6}{10}$$
$$= \frac{3}{5}$$

29. If the average of $x, y$, and 32 is 28, then the average of $x$ and $y$ would be

A. 24
B. 26
C. 28
D. 30

*By applying the formula used to calculate an average and using the information given the reading, you can write an equation and solve for $x + y$.*

$$28 = \frac{x + y + 32}{3}$$
$$3(28) = x + y + 32$$
$$84 = x + y + 32$$
$$84 - 32 = x + y$$
$$52 = x + y$$

Please Turn Over

Since the problem asks for the average of $x$ and $y$, next apply the formula to calculate an average using $x + y = 52$:

$$average = \frac{x+y}{2}$$
$$= \frac{52}{2}$$
$$= 26$$

30. Simplify $\sqrt[4]{27x^2\sqrt{9x^4}}$

A. $3\sqrt[4]{x}$
B. $3\sqrt{x}$
C. $3x$
D. $\sqrt{3x}$

*To solve this problem start with the innermost radical and work out using the rules of exponents and the properties of radicals:*

$$\sqrt[4]{27x^2\sqrt{9x^4}}$$

$$\sqrt[4]{27x^2\sqrt{9}\sqrt{x^4}}$$

$$\sqrt[4]{27x^2\,3x^2}$$

$$\sqrt[4]{27x^2\,3x^2}$$

$$\sqrt[4]{27}\,\sqrt[4]{x^2}\,\sqrt[4]{3}\,\sqrt[4]{x^2}$$

$$\sqrt[4]{27}\,\sqrt[4]{3}\,\sqrt[4]{x^2}\,\sqrt[4]{x^2}$$

$$\sqrt[4]{27\cdot 3}\,\sqrt[4]{x^2\cdot\,x^2}$$

$$\sqrt[4]{3^3\cdot 3}\,\sqrt[4]{x^4}$$

$$\sqrt[4]{3^4}\cdot x$$

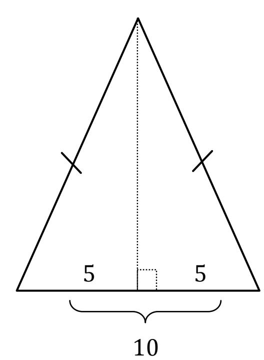
$$3x$$

31. An isosceles triangle with base 10 cm has an area of 60 cm². Find its perimeter.

A. 23 cm
B. 26 cm
C. 36 cm
D. 41 cm

*Start by drawing a diagram of the triangle described in the reading. Remember, an isosceles triangle has two congruent (identical or equal) sides. The height bisects the base and divides the original triangle into two right triangles.*

Please Turn Over



*Then use the given area to find the height:*

$$area = \frac{1}{2}bh$$
$$60 = \frac{1}{2}(10)h$$
$$120 = 10h$$
$$12 = h$$

*Then use the height and the Pythaorean Theorem to calculate the length of the remaining sides:*

$$a^2 + b^2 = c^2$$
$$12^2 + 5^2 = c^2$$
$$144 + 25 = c^2$$
$$169 = c^2$$
$$\sqrt{169} = \sqrt{c^2}$$
$$13 = c$$

*So the two identical sides measure 13cm. Finaally calculate the perimeter by adding the lengths of the three sides:*

$$perimeter = side_1 + side_2 + side_3$$
$$= 10 + 13 + 13$$
$$= 36\ cm$$

32. If the graph of $y = 2(x-1)^2 + k$ passes through the point $(3, -7)$, then $k$ equals?

    A. $-15$
    B. $-1$
    C. $1$
    D. $15$

*To solve this problem substitute $x = 3$ and $y = -7$ into the equation and solve for $k$. Remember, points are always written in the form $(x, y)$.*

$$y = 2(x-1)^2 + k$$
$$-7 = 2(3-1)^2 + k$$
$$-7 = 2(2)^2 + k$$
$$-7 = 2(4) + k$$
$$-7 = 8 + k$$
$$-7 - 8 = k$$
$$-15 = k$$

33. Solve the following:

$$\frac{a}{a-b} + \frac{b}{b-a}$$

    A. $-1$
    B. $\frac{a+b}{a-b}$
    C. $1$
    D. $\frac{b+a}{b-a}$

*Remember, fractions can only be added to each other if they have the same denominator. Notice: you can rewrite the second denominator by rearranging the variables and then factoring out $-1$.*

$$\frac{a}{a-b} + \frac{b}{b-a}$$
$$\frac{a}{a-b} + \frac{b}{-a+b}$$
$$\frac{a}{a-b} + \frac{b}{-(a-b)}$$
$$\frac{a}{a-b} - \frac{b}{a-b}$$
$$\frac{a-b}{a-b}$$
$$1$$

34. Find the sum of 14 terms of $-8, -3, 2, 7, \ldots$

A. 57
B. 77
C. 343
D. 378

Step #1 FIND THE PATTERN

*Look at each term of the sequence and examine its relationship to the terms directly before and after it. In this problem you notice that 5 is added to each term to obtain the next term in the sequence:*

$$\overbrace{-8, -3,}^{-8+5=-3}\ 2, 7 \ldots$$

$$-8, \overbrace{-3, 2,}^{-3+5=2}\ 7, \ldots$$

$$-8, -3, \overbrace{2, 7,}^{2+5=7}\ \ldots$$

Step #2 FIND THE TERMS

*The first four terms are given so you need to generate ten more:*

$-8, -3, 2, 7, 13, 18, 23, 28, 33, 38, 43, 48, 53, 58, \ldots.$

Step #3 ADD THE TERMS

*The word 'sum' tells you to do addition. So take all 14 terms and add them up:*

$$-8 - 3 + 2 + 7 + 13 + 18 + 23 + 28 + 33 + 38 + 43 + 48 + 43 + 58 = 343$$

35. If $M$ is the midpoint of $\overline{AB}$ and $M(-4.2, -1.9)$, $A(-3.4, 2.5)$ are given, then $\mathbf{B}$ is?

A. $(-3.8, -0.6)$
B. $(-5, -6.3)$
C. $(-3.8, 0.3)$
D. $(1.3, -3.45)$

*Midpoint problems are very simple to solve if you know the formula—so memorize it!*

$$midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

*where $(x_1, y_1)$ and $(x_2, y_2)$ are endpoints of a line segment.*

*You are given the midpoint $M$ and endpoint $A$ and are asked to find the other endpoint $B$. So write:*

$$(-4.2, -1.9) = \left(\frac{-3.4 + x_2}{2}, \frac{2.5 + y_2}{2}\right)$$

*To make things easier, you can break this into two smaller algebra problems:*

$$-4.2 = \frac{-3.4 + x_2}{2} \qquad -1.9 = \frac{2.5 + y_2}{2}$$
$$(-4.2)(2) = -3.4 + x_2 \qquad (-1.9)(2) = 2.5 + y_2$$
$$-8.4 = -3.4 + x_2 \qquad -3.8 = 2.5 + y_2$$
$$-5 = x_2 \qquad -6.3 = y_2$$

*This means the coordinates of $B = (-5, -6.3)$*

36. John left a certain amount of money to be shared among his children: Esther, Mary, and Ruth in the ratio of 2:3:4. If Mary received $210.00, how much did Esther receive?

A. $280.00
B. $140.00
C. $70.00
D. $46.67

*You are given that*

$$Esther: Mary: Ruth = 2:3:4 = 2x:210:4x$$

*Since Mary gets $210 it follows that:*

$$3x = 210$$

$$x = \frac{210}{3}$$

$$x = 70$$

*Then Esther receives* $2x = 2(70) = 140$ //

37. Find the values of $x$ in $112_x = 22$

A. $4\ and\ 5$
B. $4\ and\ -5$
C. $-4\ and\ -5$
D. $-4\ and\ 5$

*This problem asks you to use knowledge of* ***Number Bases*** *to solve when the base is unknown.*

Step #1 *CREATE A TABLE*

*Focus first on the left side of the equation. Convert* $112_x$ *to a base 10 number by creating a table. There should always be two rows and then one column for each digit in the number. For example, in this problem we need three columns.*

*In the top row write powers of the base with the exponents decreasing from left to right, always ending in zero. In the bottom row write one digit in each column. This essentially matches each digit with its place value in the base.*

| $x^2$ | $x^1$ | $x^0$ |
|---|---|---|
| *1* | *1* | *2* |

Step #2 *MULTIPLY AND COMBINE*

*Multiply the contents of each column and add the results*

$$(1)(x^2) = x^2$$
$$(1)(x^1) = x$$
$$\underline{(2)(x^0) = 2}$$

$$= x^2 + x + 2$$

Step #3 *SOLVE*

*Set this equation equal to the right side of the original equation and solve for* $x$

$$x^2 + x + 2 = 22$$
$$x^2 + x + 2 - 22 = 0$$
$$x^2 + x - 18 = 0$$
$$(x-4)(x+5) = 0$$
$$x = 4\ and\ -5$$ //

38. Given that

$U = \{1,2,3,4,5,6,7,9,11,13,17,19\}$
$A = \{1,2,3,4,5,6,7\}, and$
$B = \{1,3,5,7,9,11\}, find\ A' \cap B'$

A. {9,11,13,17,19}
B. {2,4,6,13,17,19}
C. {13,17,19}
D. {11,13,17,19}

*Recall that when you see {braces} you know you are working with sets, or groups. In order to solve this problem, you need to know two definitions from set theory.*

*First, the set U is always defined as the* ***Universal Set****. The Universal Set is the largest set and contains everything that* <u>*could*</u> *be an element of a set within the context of the problem. In this problem:*

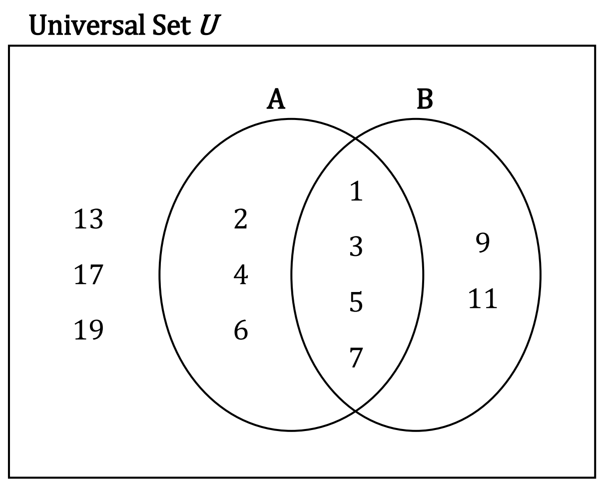
$$U = \{1,2,3,4,5,6,7,9,11,13,17,19\} = The\ Universal\ Set$$

*The numbers that are members of the universal set are the only numbers you will consider when solving this problem.*

*The second thing you need to know is that when a set A is written with an apostrophe $A'$ it means the* ***complement*** *of A. The complement of a set is everything that is in the universal set but* <u>*not*</u> *in that particular set. So $A'$ means all numbers in U but not in A.*

*Then $A' \cap B'$ means the* ***intersection of the complement of A and the complement of B.*** *It includes all numbers that are in U but are not in A or B.*

*If we draw a diagram the problem looks like this:*




*So using our definitions and looking at our diagram you can write:*

$$A' = \{9,11,13,17,19\}$$
$$B' = \{2,4,6,13,17,19\}$$

*$A' \cap B'$ asks us to find the elements $A'$ and $B'$ share so:*

$$A' \cap B' = \{13,17,19\}$$

39. Solve for x in the equation:

$$\frac{3}{4}(x-2)+\frac{1}{2}=x-1$$

A. $-4$
B. 0
C. 1.5
D. 2

*Use algebra to solve for x:*

$$\frac{3}{4}(x-2)+\frac{1}{2}=x-1$$
$$\frac{3}{4}(x-2)=x-1-\frac{1}{2}$$
$$\frac{3}{4}(x-2)=x-\frac{3}{2}$$
$$4\left(\frac{3}{4}(x-2)\right)=4\left(x-\frac{3}{2}\right)$$
$$3(x-2)=4x-\frac{12}{2}$$
$$3x-6=4x-6$$
$$3x=4x$$
$$0=4x-3x$$
$$0=x$$

40. What is the solution set of $10-|2x-9|=7$?

A. $-6$ *and* $-3$
B. $-6$ *and* $3$
C. $-3$ *and* $6$
D. $3$ *and* $6$

*Recall that when you see* $|straight\ brackets|$ *this refers to absolute value, or the distance from zero of any number on the number line (and distance is always positive). By definition, absolute value problems need to be solved twice and often result in two correct answers.*

Step #1

*Simplify the equation so the absolute value is isolated on one side.*

$$10-|2x-9|=7$$
$$-|2x-9|=-3$$
$$|2x-9|=3$$

Step #2

*Replace the straight brackets with parenthesis and solve the equation twice. First, taking the absolute value to be a positive expression, then taking it to be a negative expression. Solve for* $x$ *in both equations.*

| | |
|---|---|
| $(2x-9)=3$ | $-(2x-9)=3$ |
| $2x-9=3$ | $-2x+9=3$ |
| $2x=12$ | $-2x=-6$ |
| $x=6$ | $x=3$ |

*So given* $|2x-9|=3$ *it follows that* $x=3$ *and* $6$

41. Find the value of x in the below equation:

$$3x + \begin{bmatrix} -9 & 2 \\ 1 & -5 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 & -3 \\ -6 & 12 \end{bmatrix}$$

A. $\begin{bmatrix} -4 & -1 \\ -1 & 3 \end{bmatrix}$

B. $\begin{bmatrix} 4 & -1 \\ -1 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 4 & -1 \\ 1 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 12 & -3 \\ -3 & 9 \end{bmatrix}$

*You can see that x will be a 2x2 matrix from the answer choices so write:*

$$x = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$$

$$3\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} + \begin{bmatrix} -9 & 2 \\ 1 & -5 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 & -3 \\ -6 & 12 \end{bmatrix}$$

*Then you match the elements of the matrices to write and solve four different equations:*

$$3x_1 - 9 = \frac{1}{3}(9)$$
$$3x_1 - 9 = 3$$
$$3x_1 = 12$$
$$x_1 = 4$$

$$3x_2 + 2 = \frac{1}{3}(-3)$$
$$3x_2 + 2 = -1$$
$$3x_2 = -3$$
$$x_2 = -1$$

$$3x_3 + 1 = \frac{1}{3}(-6)$$
$$3x_3 + 1 = -2$$
$$3x_3 = -3$$
$$x_3 = -1$$

**Note that by process of elimination you can already see the answer will be B. For the sake of good understanding we'll continue with the last equation, but on the exam you can stop to save time.*

$$3x_4 - 5 = \frac{1}{3}(12)$$
$$3x_4 - 5 = 4$$
$$3x_4 = 9$$
$$x_4 = 3$$

$$x = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -1 & 3 \end{bmatrix}$$

42. The two geometric means between 4 and 108 are?

A. $-12$ and $-36$
B. $-36$ and $12$
C. $-12$ and $36$
D. $12$ and $36$

Short Cut! Geometric means only deal with positive numbers. If you remember this you can immediately eliminate all answer choices but D.

*Take time not to confuse* **geometric mean** *with the more common* **arithmetic mean**. *What this problem asks you to do is find two numbers between 4 and 108 such that the ratio of consecutive numbers is the same.*

*You have a sequence of four numbers* $a_1, a_2, a_3, a_4$ *where* $a_1 = 4$, $a_4 = 108$ *and* $a_2$ *and* $a_3$ *are the two geometric means in between.*

$$\frac{a_1}{a_2} = \frac{a_2}{a_3} = \frac{a_3}{a_4}$$

$$\frac{4}{a_2} = \frac{a_2}{a_3} = \frac{a_3}{108}$$

*If you cross multiply the equations you arrive at the simultaneous equations:*

$$\begin{cases} 4a_3 = (a_2)^2 \\ 108a_2 = (a_3)^2 \end{cases}$$

*This can be solved using substitution. Solve the first equation for* $a_3$ *then substitute the result into the second equation:*

$$4a_3 = (a_2)^2$$

$$a_3 = \frac{(a_2)^2}{4}$$

$$108a_2 = (a_3)^2$$

$$108a_2 = \left(\frac{(a_2)^2}{4}\right)^2$$

$$108a_2 = \frac{(a_2)^4}{4^2}$$

$$108a_2 = \frac{(a_2)^4}{16}$$

$$16(108a_2) = (a_2)^4$$

$$1{,}728a_2 = (a_2)^4$$

$$1{,}728 = \frac{(a_2)^4}{a_2}$$

$$1{,}728 = (a_2)^3$$

$$\sqrt[3]{1{,}728} = \sqrt[3]{(a_2)^3}$$

$$12 = a_2$$

Please Turn Over

*Then substitute* $a_2 = 12$ *back into the first equation and solve for the value of* $a_3$*:*

$$4a_3 = (a_2)^2$$

$$4a_3 = (12)^2$$

$$4a_3 = 144$$

$$a_3 = \frac{144}{4}$$

$$a_3 = 36$$

*Return to the original equation and substitute the values of* $a_1, a_2, a_3, a_4$ *to check:*

$$\frac{a_1}{a_2} = \frac{a_2}{a_3} = \frac{a_3}{a_4}$$

$$\frac{4}{12} = \frac{12}{36} = \frac{36}{108}$$

$$\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$$

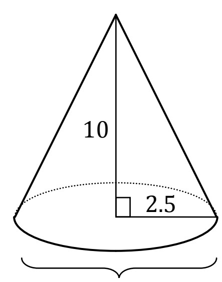
*So the two geometric means between 4 and 108 are 12 and 36.* //

43. A circular based pyramid has a height of 10cm and a diameter of 5cm. What could be the volume of this pyramid?

    ($Note: Volume\ of\ pyramid = \frac{1}{3} \times base\ area \times height$. Use π=3.14

    A. 785cm³
    B. 654cm³
    C. 261.7cm³
    D. 65.4cm³

**Step #1** *Draw a diagram of the figure described in the reading.*




**Step #2** *Substitute the values given to find the area of the base then the volume.*

$$base\ area = \pi r^2$$
$$= \pi(2.5cm)^2$$
$$= 6.25\pi\ cm^2$$

$$volume = \frac{1}{3} \times base\ area \times height$$
$$= \frac{1}{3}(6.25\pi\ cm^2)(10\ cm)$$
$$= \frac{1}{3}(6.25\ cm^2)(3.14)(10\ cm)$$
$$= 65.4cm^3$$ //

44. A line with slope $\frac{-5}{2}$ passes through points $(7,-4)$ and $(x,6)$. What is the value of x?

A. 11
B. 3
C. $-3$
D. $-11$

*If $(x_1,y_1)$ and $(x_2,y_2)$ are points on a line then the slope of the line is given by the equation:*

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

*Let $(x_1,y_1) = (7,-4)$ and $(x_2,y_2) = (x,6)$. Then you can substitute and solve:*

$$-\frac{5}{2} = \frac{-4-6}{7-x}$$

$$-\frac{5}{2} = \frac{-10}{7-x}$$

$$2(-10) = -5(7-x)$$

$$-20 = -35 + 5x$$

$$-20 + 35 = 5x$$

$$15 = 5x$$

$$3 = x$$

45. If the circumference of a circle is 25.12cm, find the area of the circle. (use $\pi = 3.14$)

A. 5.02cm²
B. 50.24cm²
C. 200.96cm²
D. 502.4cm²

*To solve this problem you need to know two formulas about circles.*

$$Circumference = 2\pi \times radius$$
$$Area = \pi \times (radius)^2$$

Step #1 *Use the circumference (which is given) to find the radius:*

$$C = 2\pi r$$
$$25.12 = 2(3.14)r$$
$$25.12 = (6.28)r$$
$$\frac{25.12}{6.28} = r$$
$$4 = r$$

Step #2 *Use the radius to calculate the area:*

$$A = \pi r^2$$
$$= \pi(4^2)$$
$$= 16\pi$$
$$= 16(3.14)$$
$$= 50.24cm^2$$

46. Six boys and 3 girls are eligible for a 5 member team. In how many ways can the team be formed with exactly 3 boys?

A. 720
B. 100
C. 60
D. 23

*When a problem asks you to choose objects and the order they are selected in does <u>not</u> matter, like when choosing members for a team, you use **combinations:**:*

$$_nC_r = \frac{n!}{r!\,(n-r)!}$$

*where n is the total number of things chosen r at a time. This is read "n choose r." Remember that the ! indicates that you take the **factorial**.*

Step #1 *Calculate the total possible combinations that can be created by choosing 3 of the 6 boys:*

$$\begin{aligned} {}_6C_3 &= \frac{n!}{r!\,(n-r)!} \\ &= \frac{6!}{3!\,(6-3)!} \\ &= \frac{6!}{3!\,3!} \\ &= \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{(3\cdot2\cdot1)(3\cdot2\cdot1)} \\ &= \frac{6\cdot5\cdot4\cdot\cancel{3\cdot2\cdot1}}{(3\cdot2\cdot1)(\cancel{3\cdot2\cdot1})} \\ &= \frac{6\cdot5\cdot4}{3\cdot2\cdot1} \\ &= \frac{120}{6} \\ &= 20 \end{aligned}$$

Step #2 *Calculate the total possible combinations that can be created by choosing 2 of the 3 girls:*

$$\begin{aligned} {}_3C_2 &= \frac{3!}{2!\,(3-2)!} \\ &= \frac{3!}{2!\,(1)!} \\ &= \frac{3\cdot2\cdot1}{2\cdot1\cdot1} \\ &= 3 \end{aligned}$$

Step #3 *Multiply the total possible combinations of girls and the total possible combinations of boys:*

$$\begin{aligned} total\ combinations &= C_{boys} \times C_{girls} \\ &= 20\ \times 3 \\ &= 60 \end{aligned}$$

47. A bag of 30 fruits contains oranges, mangoes, and grapes. The probability of picking an orange is $\frac{1}{2}$. If there are 5 mangoes, find the probability that a fruit picked at random is an orange or a grape.

A. 1/3
B. 5/6
C. 1/2
D. 1/6

*This problem gives you more information that you need. We can ignore the probability of choosing an orange and use instead that there are 5 mangoes in the bag.*

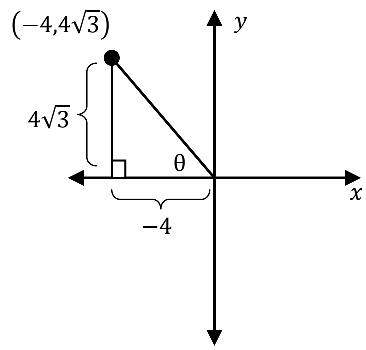
$$\begin{aligned} total\ fruit &= oranges + grapes + mangoes \\ 30 &= oranges + grapes + 5 \\ 30 - 5 &= oranges + grapes \\ 25 &= oranges + grapes \end{aligned}$$

$$\begin{aligned} P(orange\ or\ grape) &= \frac{\#\ oranges\ and\ grapes}{total\ \#\ of\ fruit} \\ &= \frac{25}{30} \\ &= \frac{5}{6} \end{aligned}$$

48. The terminal side of an angle contains the point $(-4, 4\sqrt{3})$. The cosine of this angle is?

A. $\frac{\sqrt{3}}{2}$
B. $-1/2$
C. $-\sqrt{3}$
D. $-2$

Step #1 *Draw a diagram of the problem, situating it on a coordinate plane and recalling that points are written in the form* $(x, y)$.




Step #2 *Now, draw a line from the point straight down to the x-axis (parallel to the y-axis) and a second line from the point to the origin. Label the length of each side of the triangle that is formed.*

*The length of the two known sides suggests a 30·60·90* ***special right triangle*** *with its sides related by* $\ell \cdot \ell\sqrt{3} \cdot 2\ell$. *So it follows that:*

$$\begin{aligned} hypotenuse &= 2\ell \\ &= 2(4) \\ &= 8 \end{aligned}$$

If you don't recognize the special right triangle you can easily solve for the length of the hypotenuse using the Pythagorean Theorem: $a^2 + b^2 = c^2$.

Step #3 *Substitute the known side-lengths from the problem into the formula for the cosine of an angle and solve:*

$$\begin{aligned} \cos\theta &= \frac{adjacent}{hypotenuse} \\ &= -\frac{4}{8} \\ &= -\frac{1}{2} \end{aligned}$$

49. Simplify

$$\frac{c^2 - cd}{d^2 - de} \div \frac{d^2 - cd}{cd - ce}$$

A. $c^2/d^2$
B. $d^2/c^2$
C. $-c^2/d^2$
D. $-d^2/c^2$

*Remember, dividing by a fraction can be rewritten and solved using multiplication instead of division. Simply multiply by the reciprocal (inverse) of the fraction.*

$$\frac{c^2 - cd}{d^2 - de} \div \frac{d^2 - cd}{cd - ce} = \frac{c^2 - cd}{d^2 - de} \cdot \frac{cd - ce}{d^2 - cd}$$

$$= \frac{c(c-d)}{d(d-e)} \cdot \frac{c(d-e)}{d(d-c)}$$

$$= \frac{c(c-d)}{d(\cancel{d-e})} \cdot \frac{c(\cancel{d-e})}{d(d-c)}$$

$$= \frac{c^2(c-d)}{d^2(d-c)}$$

$$= \frac{-c^2(-c+d)}{d^2(d-c)}$$

$$= \frac{-c^2(d-c)}{d^2(d-c)}$$

$$= \frac{-c^2(\cancel{d-c})}{d^2(\cancel{d-c})}$$

$$= \frac{-c^2}{d^2}$$

50. A student scores 74%, 81%, 62%, 58% and 77% in five subjects. How much should the student score in the sixth test to make an average of 74%?

A. 96%
B. 92%
C. 89%
D. 82%

*In order to calculate the score from the sixth test, simply calculate the average for all six tests—substituting a variable for the unknown test score. Then solve and find out the value of the variable (representing the score for the sixth test).*

$$average = \frac{total\ scores\ earned}{otal\ tests\ taken}$$

$$74 = \frac{74 + 81 + 62 + 58 + 77 + x}{6}$$

$$74 = \frac{352 + x}{6}$$

$$444 = 352 + x$$

$$92 = x$$

*So the student must score a 92% on the sixth test to make an average of 74%.*

**PAPER 2** 1 ½ Hours

**ESSAY**

[60 Marks]

*Paper 2 consists of seven questions divided into two sections, A & B. Section A contains four compulsory questions and section B contains three questions from which you are required to answer any two. Write your answers in ink (blue or black) only. Credit will be given for clarity of expression and orderly presentation of material.*

*NOTE: Make sure you understand these directions! The majority of students perform poorly on this section because they do not answer enough questions. Solutions are provided here with the amount of detail and 'clarity of expression' you should include on the actual exam.*

**SECTION A**

**COMPULSORY**

[35 Marks]

*There are four questions in this section. You are required to answer all the parts of all the questions.*

1. *(a)* Find $\log_{10} 40$ given that $\log_{10} 2 = 0.3010$.

*This question requires you to use the properties of logarithms to rewrite and evaluate* $\log_{10} 40$*:*

| | |
|---|---|
| $\log_{10} 40 = \log_{10}(4 \cdot 10)$ | Factor 40 into $(4 \cdot 10)$ |
| $= \log_{10} 4 + \log_{10} 10$ | Apply Product Property |
| $= \log_{10} 2^2 + \log_{10} 10$ | Write 4 as $2^2$ |
| $= 2\log_{10} 2 + \log_{10} 10$ | Power Property |
| $= 2(0.3010) + 1$ | Substitute |
| $= 0.6020 + 1$ | Simplify |
| $= 1.6020$ // | |

*(b)* Find the value of $\boldsymbol{k}$ if $(k, 1)$ is a solution of $y = 2^x$

*Recall that ordered pairs are always written in the form* $(x, y)$*. To solve this problem all you do is substitute:*

| | |
|---|---|
| $y = 2^x$ | *Rewrite the equation* |
| $1 = 2^k$ | *Substitute* |
| $2^0 = 2^k$ | *Cancel the base 2* |
| $0 = k$ // | *Only the exponents are left* |

2. *(a)* A man is 20cm taller than his wife. The wife is 68cm taller than half the man's height. How tall is the man?

*Solve this reading problem using the same three step method use in objective problem #21:*

Step #1 *UNDERSTAND*

*What do you know?*

- *A man is 20cm taller than his wife*
- *The wife is 68cm taller than half the man's height*

*What do you want to know?*

- *How tall is the man?*

Step #2 *PLAN*

ASSIGN VARIABLES

$$w = wife's\ height$$
$$m = man's\ height$$

WRITE EQUATIONS

*Use your variables to turn each of your 'What do you know?' sentences into an equation.*

$$m = 20 + w$$

$$w = 68 + \tfrac{1}{2}m$$

Step #3 *SOLVE AND CHECK*

*Substitute the second equation into the first equation then collect like terms:*

$$m = 20 + \left(68 + \frac{1}{2}m\right)$$

$$m = \frac{1}{2}m + 88$$

$$m - \frac{1}{2}m = 88$$

$$\frac{1}{2}m = 88$$

$$m = 2(88)$$

$$m = 176cm$$

(b) Given that $12x - 8y = \frac{44}{3}$, find the value of $9x - 6y$.

*Look at the coefficients of the $x$ terms and notice that 9 is $\frac{3}{4}$ of 12. Then look at the coefficients of the $y$ terms and notice that 6 is $\frac{3}{4}$ of 8. Then you can write:*

$$9x - 6y = \frac{3}{4}(12x - 8y)$$

*Since you are given* $12x - 8y = \frac{44}{3}$, *substitute and solve:*

$$9x - 6y = \frac{3}{4}\left(\frac{44}{3}\right)$$

$$= 11$$

3\. (a) Find the remainder when $2x^4 - x^3 + 3x - 1$ is divided by $x + 2$

*This problem requires you to do long division with polynomials.*

$$\begin{array}{r|l}
 & 2x^3 - 5x^2 + 10x - 17 \\
\hline
x+2 & 2x^4 - x^3 + 0x^2 + 3x - 1 \\
 & \underline{(2x^4 + 4x^3)} \\
 & \quad -5x^3 + 0x^2 \\
 & \quad -\underline{(-5x^3 - 10x^2)} \\
 & \qquad\qquad 10x^2 + 3x \\
 & \qquad\qquad -\underline{(10x^2 + 20x)} \\
 & \qquad\qquad\qquad -17x - 1 \\
 & \qquad\qquad\qquad -\underline{(-17x - 34)} \\
 & \qquad\qquad\qquad\qquad\quad 33
\end{array}$$

*Since* $(x + 2)$ *does not go into 33 you are finished. Divide 33 by the divisor to get the answer:*

$$\frac{33}{x + 2}$$

(*b*) Solve the system of equations for $x$ and $y$:

$$\begin{cases} 3(x+y) = 7(y-x) \\ 5(3x-y) = x+3 \end{cases}$$


*Good mathematicians always check their work. If you have time you should substitute your values for $x$ and $y$ into the original equations to make sure they are solutions of both equations.*

*The solution to this system will be a pair of $x$ and $y$ values that make both equations true statements.*

Step #1

*Solve one equation for one of the variables. It does not matter which you choose:*

$$\begin{aligned} 3(x+y) &= 7(y-x) \\ 3x+3y &= 7y-7x \\ 3y &= 7y-10x \\ -4y &= -10x \\ y &= \frac{-10x}{-4} \\ y &= \frac{5x}{2} \end{aligned}$$

Step #2

*Substitute this result into the second equation then collect like terms and solve for $x$.*

$$5(3x-y) = x+3$$

$$5\left(3x-\frac{5x}{2}\right) = x+3 \qquad \textit{Substitute } y = \tfrac{5x}{2}$$

$$15x-\frac{25x}{2} = x+3 \qquad \textit{Distribute the 5}$$

$$\frac{30x}{2}-\frac{25x}{2} = x+3 \qquad \textit{Get a common denominator}$$

$$\frac{5x}{2} = x+3 \qquad \textit{Subtract}$$

$$5x = 2(x+3) \qquad \textit{Multiply the 2 across}$$

$$5x = 2x+6 \qquad \textit{Distribute the 2}$$

$$3x = 6 \qquad \textit{Subtract the } 2x \textit{ across}$$

$$x = 2 \qquad \textit{Divide all by 3}$$

Step #3

*Substitute that $x$ value back into step 1:*

$$y = \frac{5x}{2}$$

$$y = \frac{5(2)}{2}$$

$$y = 5$$

*So the solution to the system is $x = 2$ and $y = 5$* //

4. *(a)* Solve for $x$ in $7^x = \frac{49}{\sqrt[3]{7}}$

*This problem requires you to apply the rules of exponents to manipulate and simplify the equation.*

$$7^x = \frac{49}{\sqrt[3]{7}}$$

$$7^x = \frac{7^2}{\sqrt[3]{7}}$$ *Rewrite 49 as a power of 7*

$$7^x = \frac{7^2}{7^{\frac{1}{3}}}$$ *Rewrite the cube root as an exponent*

$$7^x = (7^2)(7^{-\frac{1}{3}})$$ *Negative Exponent Rule*

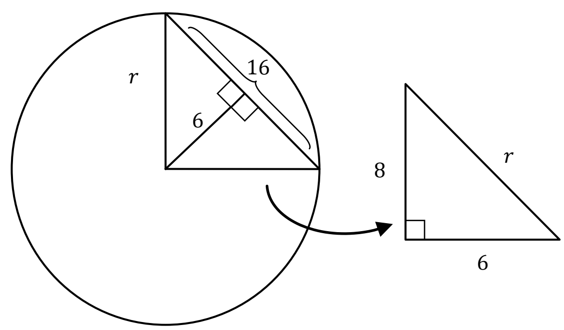
$$7^x = 7^{(2-\frac{1}{3})}$$ *Product Rule*

$$7^x = 7^{\frac{5}{3}}$$ *Subtract*

$$x = \frac{5}{3}$$

*(b)* The chord of a circle is 16cm long. If it is 6cm from the center of the circle, find the radius of this circle.

*Given a problem like this your first step should always be to draw a diagram.*




*Then you can easily solve using right triangles and the Pythagorean Theorem:*

$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = r^2$$
$$36 + 64 = r^2$$
$$100 = r^2$$
$$10 = r$$

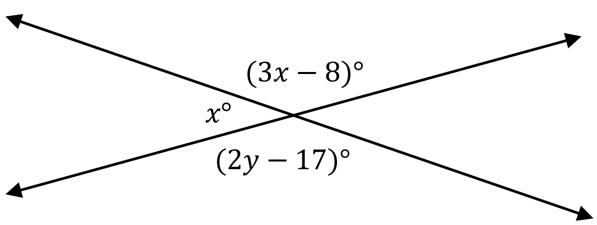
*So the radius of the circle is* 10cm

## SECTION B

[24 Marks]

*Section B contains three questions. You are required to answer all the parts of any **two** of the three questions.*

5. From the given diagram below, find the




*(a)* Measure of ∠x.

*(b)* Measure of $\angle(3x-8)$.

*(c)* Value of $y$.

*These three problems require you to use your knowledge of angles to write and solve simple equations.*

*(a)* *By definition a straight line is 180° so write:*

$$x+(3x-8)=180$$

$$x+3x-8=180$$

$$4x-8=180$$

$$4x=188$$

$$x=\frac{188}{4}$$

$$x=47°$$

*(b)* *Substituting the value of $x$ solved for above:*

$$\angle(3x-8)=(3(47)-8)$$

$$=141-8$$

$$=133°$$

*(c)* *Again using the definition of a straight line and the $x$ value solved for above:*

$$x+(2y-17)=180$$

$$47+(2y-17)=180$$

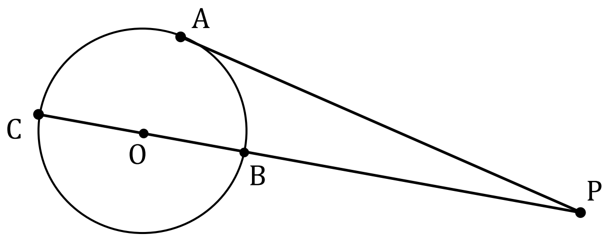
$$47+2y-17=180$$

$$2y+30=180$$

$$2y=150$$

$$y=75$$

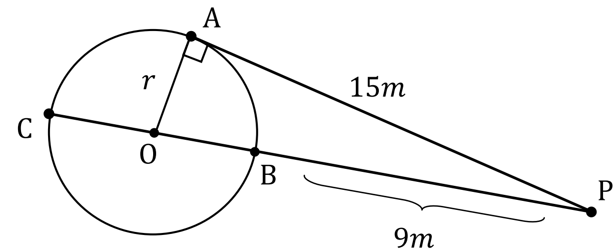
6. In the diagram below, $\overline{PA}$ is a tangent to circle $O$ at $A$ and secant $\overline{PC}$ passes through $O$. If $\overline{PA} = 15m$ and $\overline{PB} = 9m$, find the




*(a)* Radius of the circle

*(b)* Length of secant $\overline{PC}$

*Given a problem like this you should always redraw the diagram into your solution and add the additional information given in the reading.*




*(a) By connecting $A$ and $O$ you can see that a right triangle is formed by $\overline{PA}$ and $\overline{PC}$. Then by the Pythagorean Theorem it follows that:*

$$r^2 + 15^2 = (r+9)^2$$
$$r^2 + 225 = (r+9)(r+9)$$
$$r^2 + 225 = r^2 + 18r + 81$$
$$225 = 18r + 81$$
$$144 = 18r$$
$$8 = r$$

*So the radius is* $8cm.$

*(b) It is clear from the diagram that*

$$\overline{PC} = \overline{PB} + \overline{BO} + \overline{OC}$$

*Then since we know that the radius is 8m and $\overline{PB}$ is 9m we can use the definition of radius to write:*

$$\overline{PC} = 9m + 8m + 8m$$
$$= 25m$$

7. *(a)* Find the value of $x$ for which

$$(x-2)^2 : (x+3)^2 = 1:4$$

*This problem uses ratios. Remember that a ratio is just another way to express division. So you can write:*

$$\frac{(x-2)^2}{(x+3)^2} = \frac{1}{4}$$ *Rewrite as division*

$$4(x-2)^2 = (x+3)^2$$ *Cross multiply*

$$4(x^2-4x+4) = x^2+6x+9$$ *Expand*

$$4x^2-16x+16 = x^2+6x+9$$ *Distribute*

$$3x^2-16x+16 = 6x+9$$ *Combine $x^2$ terms*

$$3x^2-22x+16 = 9$$ *Combine $x$ terms*

$$3x^2-22x+7 = 0$$ *Combine constants*

$$(3x-1)(x-7) = 0$$ *Factor*

*Any number times zero is equal to zero so when either expression is equal to zero the entire equation is equal to zero.*

$$3x-1=0 \qquad\qquad x-7=0$$

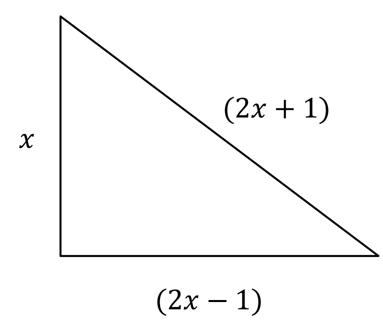
$$3x=1 \qquad\qquad x=7$$

$$x=\frac{1}{3}$$

*So* $x=\frac{1}{3}$ *or* $x=7$ //

*(b)* A triangle has sides of length $x$cm, $(2x-1)$cm and $(2x+1)$cm. If its perimeter is 40cm, find the probability that an ant, which continuously crawls around the perimeter of the triangle at a steady speed, is on the shortest side at any given moment.

*Sketch a diagram that is not to scale to help you visualize the problem:*




Please Turn Over

Step #1

*Solve for* $x$

*The perimeter of the triangle is the sum of the three side lengths so:*

$$x + (2x - 1) + (2x + 1) = 40$$
$$x + 2x - 1 + 2x + 1 = 40$$
$$5x = 40$$
$$x = 8$$

Step #2

*Find the probability of being on the shortest side. The probability the ant is on the shortest side can be expressed as:*

$$\frac{length\ of\ shortest\ side}{total\ perimeter} = \frac{8}{40} = \frac{1}{5}$$

*So the probability the ant is on the shortest side at any given moment is* $\frac{1}{5}$ //

# END OF EXAM

# It's Good to Know

**Distance Formula**
If $(x_1, y_1)$ and $(x_2, y_2)$ are points on the line:

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Mid-Point Formula**
If $(x_1, y_1)$ and $(x_2, y_2)$ are points on the line:

$$midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

**Slope of a Line**
If $(x_1, y_1)$ and $(x_2, y_2)$ are points on the line:

$$slope = \frac{y_1 - y_2}{x_1 - x_2}$$

**Slope-Intercept Form of a Line**
Let $m = slope$ and $b = y - intercept$:

$$y = mx + b$$

**Point-Slope Form of an Equation**
Let $m = slope$ and $(x_1, y_1)$ be a point on the line:

$$y - y_1 = m(x - x_1)$$

**Quadratic Formula**
Let $a, b, c$ correspond to coefficients in the general form of a quadratic equation: $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Sum and Difference of Cubes**

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

**Simple Interest**
Let $I = amount\ of interest$, $P = amount\ of\ principle$, $R = interest\ rate$, and $T = time\ in\ years$:

$$I = PRT$$

**Factorial**
Let $n$ be an integer:

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$$
$$Example: 4! = 4 \cdot 3 \cdot 2 \cdot 1$$

**Number of Permutations**
A permutation is a selection of items in which order matters. The number of permutations of $n$ items of a set arranged $r$ items at a time is:

$$_nP_r = \frac{n!}{(n-r)!}$$

**Number of Combinations**
A combination is a selection of items in which order does *not* matter. The number of combinations of $n$ items of a set chosen $r$ items at a time is:

$$_nC_r = \frac{n!}{r!\,(n-r)!}$$

**PROPERTIES OF EXPONENTS**

| | |
|---|---|
| Product Rule | $x^m \cdot x^n = x^{m+n}$ |
| Power Rule | $(x^m)^n = x^{mn}$ |
| Quotient Rule | $\frac{x^m}{x^n} = x^{m-n}$ |
| Parentheses Rule | $(ab)^m = a^m b^m$ |
| Fraction Rule | $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ |
| Negative Exponent Rule | $x^{-m} = \frac{1}{x^m}$ |
| Zero Exponent Rule | $x^0 = 1\ for\ x \neq 0$ |
| Fraction Exponent Rule | $x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$ |

**PROPERTIES OF RADICALS**

| | |
|---|---|
| Product Property | $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ |
| Quotient Property | $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ |

**PROPERTIES OF LOGARITHMS**

| | |
|---|---|
| Product Property | $\log_b MN = \log_b M + \log_b N$ |
| Quotient Property | $\log_b \frac{M}{N} = \log_b M - \log_b N$ |
| Power Property | $\log_b M^x = x \log_b M$ |

**Perimeter**
The distance around the outside of a rectangle with $l = length$ and $w = width$:

$$P = 2l + 2w \text{ or } P = 2(l + w)$$

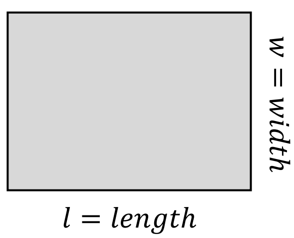
**Circumference**
The distance around the outside of a circle with $r = radius$ and $d = diameter$:

$$C = 2\pi r \text{ or } C = \pi d$$

**Area**
The area of a figure is the space enclosed inside its sides:

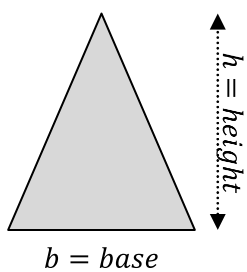
*Rectangle*

$$A = lw$$



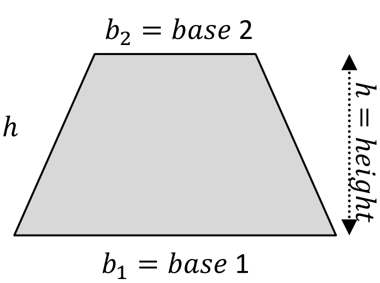

*Triangle*

$$A = \frac{1}{2}bh$$



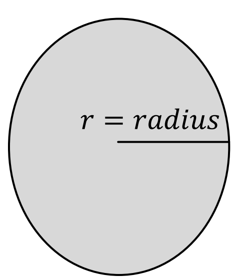

*Trapezoid*

$$A = \frac{1}{2}(b_1 + b_2)h$$



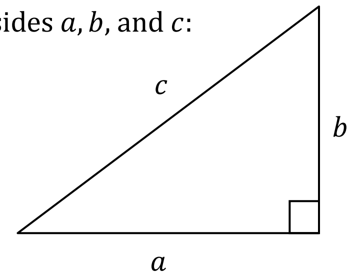

*Circle*

$$A = \pi r^2$$



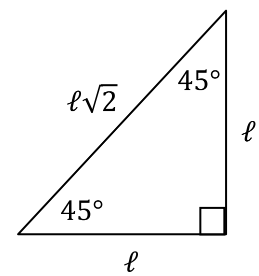

**Pythagorean Theorem**
In a right triangle with sides $a, b$, and $c$:

$$a^2 + b^2 = c^2$$

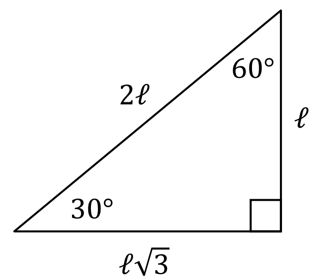



**Special Right Triangles**

$45 \cdot 45 \cdot 90$




$30 \cdot 60 \cdot 90$




**Numbers to know**

| | |
|---|---|
| 360° | • In a circle.<br>• Sum of the interior angles of a rectangle or square.<br>• Sum of the exterior angles of every polygon. |
| 180° | • In a straight line.<br>• Sum of the interior angles of a triangle. |

**Names of Polygons**

| SIDES | NAME |
|---|---|
| 3 | *Triangle* |
| 4 | *Quadrilateral* |
| 5 | *Pentagon* |
| 6 | *Hexagon* |
| 7 | *Heptagon* |
| 8 | *Octagon* |
| 9 | *Nonagon* |
| 10 | *Decagon* |
| 11 | *Undecagon* |
| 12 | *Dodecagon* |
| n | *n-gon* |

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We humbly submit our work and wish you good luck in your studies and on your exam.

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