MATH STUDY CLASS
Sanniquellie Central High
Ms. Rebekah Schulz
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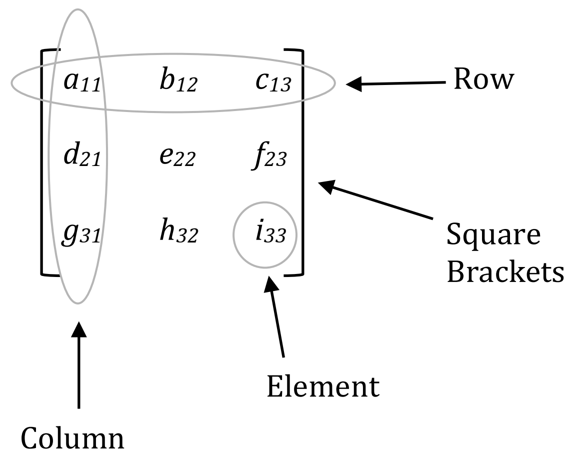
Introduction to Linear Algebra

Linear Algebra is the branch of mathematics that deals with information organized in matrices. It is very important in many fields, including computer science, information and coding theory, cryptography, chemistry, genetics, and economics.

DEF: A **matrix** (plural: matrices) organizes information, numbers, and data in rows and columns. It has four major parts.

PARTS OF A MATRIX

1. Square brackets
2. Elements
3. Rows
4. Columns



HISTORY *British-born mathematician James Joseph Sylvester first used the term "matrix" in 1850. Latin for "womb," he used it to describe a rectangular formation of numbers from which determinants could be found. (We'll talk about determinants in a few weeks.)*

A matrix can have any number of rows and columns, written in the form $row \times column$. For example, a matrix with two rows and three columns would be written 2×3, which is read "two by three." There are three types of matrices that have special names:

Row Matrix
Has only one row

$$\begin{bmatrix} a & b & c & d \end{bmatrix}$$

Column Matrix
Has only one column

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Square Matrix
Has the same number of rows and columns

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Matrices can be used to organize information and solve problems much like a spreadsheet in Microsoft Excel. This is called using **matrix logic**.

PRACTICE Use matrix logic to solve.

1. On Saturday Princess sold the following bowls of food: 15 GB with dry meat, 8 GB with fish, 16 rice with dry meat, and 12 rice with fish. She sold one bowl of dry rice with no meat. Organize the information in a 2×3 matrix.
   a. How many bowls of rice were sold?
   b. How many bowls of GB were sold?
   c. How many customers took dry meat?
   d. How many bowls customers took fish?

2. A certain NGO visits their sites in the field using both motorbikes and cars. One week they visited Kahnplay, Yekepa, Ganta, and Sanniquellie. At the end of the week they noticed some reports were missing. Organize the information from the chart in a 2×4 matrix and help the team leader complete her report.

	CAR	MOTORBIKE
Kahnplay	3	?
Yekepa	?	2
Ganta	1	2
Sanniquellie	3	1

NOTES…
7 total trips were made by motorbike
4 trips were made to Yekepa

RULE Two matrices are equal if and only if (*iff*) they have the same dimensions and the identical elements appear in the identical positions.

$$\begin{bmatrix} 4 & -7 & 12 \end{bmatrix} \neq \begin{bmatrix} 4 \\ -7 \\ 12 \end{bmatrix}$$

NOT EQUAL MATRICES
Identical elements but different positions

$$\begin{bmatrix} 3 & 8 \\ 24 & -3 \end{bmatrix} \neq \begin{bmatrix} 5 & 8 \\ 24 & -3 \end{bmatrix}$$

NOT EQUAL MATRICIES
Identical dimensions but different elements

$$\begin{bmatrix} 7 & 3 \\ 18 & -4 \\ 6 & 11 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 18 & -4 \\ 6 & 11 \end{bmatrix}$$

EQUAL MATRICIES
Identical elements in identical positions

You can use the definition of equal matrices to solve for variables when elements of the matrices are algebraic expressions.

EXAMPLE Solve for each variable: $\begin{bmatrix} 4+a \\ 2b-3 \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$

Since there is an equal sign you know they are equal matrices. That means their corresponding elements are equivalent. Write and solve two linear equations:

$$\begin{aligned} 4 + a &= 7 \\ -4 \quad &\;\; -4 \\ \hline a &= 3 \end{aligned}$$

$$\begin{aligned} 2b - 3 &= 9 \\ +3 \quad &\;\; +3 \\ \hline 2b &= 12 \\ \frac{2b}{2} &= \frac{12}{2} \\ b &= 6 \end{aligned}$$

check

$$\begin{aligned} 4 + a &= 7 \\ 4 + 3 &= 7 \\ 7 &= 7 \end{aligned}$$

check

$$\begin{aligned} 2b - 3 &= 9 \\ 2(6) - 3 &= 9 \\ 12 - 3 &= 9 \\ 9 &= 9 \end{aligned}$$

PRACTICE Solve for each variable.

1. $\begin{bmatrix} 2x & 3 & 3z \end{bmatrix} = \begin{bmatrix} 5 & 3y & 9 \end{bmatrix}$

2. $\begin{bmatrix} 6x \\ y \end{bmatrix} = \begin{bmatrix} 62 + 8y \\ 6 - 2x \end{bmatrix}$

3. $\begin{bmatrix} 2x + y \\ x - 3y \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \end{bmatrix}$

4. $\begin{bmatrix} 5x - 7 & 11 \\ 5 & 23 \end{bmatrix} = \begin{bmatrix} 8 & 21 - m \\ r^3 - 3 & 4y + x \end{bmatrix}$

DEF: **Scalar multiplication** is the process of multiplying each element in a matrix by the same constant. This constant is called a **scalar**.

RULE To perform scalar multiplication on a matrix distribute or send the constant to each element. Then simplify.

$$k \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$$

EXAMPLE Simplify.

1. $4 \begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 4(-1) & 4(0) \\ 4(3) & 4(-2) \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 12 & -8 \end{bmatrix}$

2. $x \begin{bmatrix} 4 & y \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} x(4) & x(y) \\ x(7) & x(2) \end{bmatrix} = \begin{bmatrix} 4x & xy \\ 7x & 2x \end{bmatrix}$

PRACTICE Simplify.

1. $3\begin{bmatrix} 5 & -2 & 7 \\ -3 & 8 & 4 \end{bmatrix}$

2. $-2\begin{bmatrix} 6 & -4 \\ -2 & 4 \end{bmatrix}$

3. $2z\begin{bmatrix} 9 & 2 & -11 \\ 4 & 6 & 0 \\ 0 & 8 & 13 \end{bmatrix}$

CHALLENGE! Solve for the variables.

1. $$x\begin{bmatrix} 4 & y \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 12 & -15 \\ 21 & z \end{bmatrix}$$

2. $$4\begin{bmatrix} x & y-1 \\ 3 & z \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 6z & x+y \end{bmatrix}$$

3. $$\begin{bmatrix} x^2 & 7 & 9 \\ 5 & 12 & 6 \end{bmatrix} = \begin{bmatrix} 25 & 7 & y \\ 5 & 2z & 6 \end{bmatrix}$$

4. $$\begin{bmatrix} r^2-24 & 17 \\ 7 & t^3 \end{bmatrix} = \begin{bmatrix} 1 & 2y+3 \\ z^2-12 & 27 \end{bmatrix}$$

5. $$\begin{bmatrix} 4b+2 & -3 & 4d \\ -4a & 2 & 3 \\ 2f-1 & -14 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 2c-1 & 0 \\ -8 & 2 & 3 \\ 0 & 3g-2 & 1 \end{bmatrix}$$

6. $$\begin{bmatrix} x^2 & 4 \\ -2 & y^2 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ -2 & 5y \end{bmatrix}$$

7. $$\begin{bmatrix} 4c & 2-d & 5 \\ -3 & -1 & 2 \\ 0 & -10 & 15 \end{bmatrix} = \begin{bmatrix} 2c+5 & 4d & g \\ -3 & h & f-g \\ 0 & -4c & 15 \end{bmatrix}$$

RULE To add matrices with the same dimensions add the elements in the same positions.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \quad and \quad B = \begin{bmatrix} r & s & t \\ u & v & w \end{bmatrix}$$

$$A + B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} + \begin{bmatrix} r & s & t \\ u & v & w \end{bmatrix} = \begin{bmatrix} (a+r) & (b+s) & (c+t) \\ (d+u) & (e+v) & (f+w) \end{bmatrix}$$

EXAMPLE Find the sum.

$$\begin{bmatrix} 1 & -2 & 0 \\ 3 & -5 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 9 & -3 \\ -9 & 6 & 12 \end{bmatrix} = \begin{bmatrix} (1+3) & (-2+9) & (0+-3) \\ (3+-9) & (-5+6) & (7+12) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 7 & -3 \\ -6 & 1 & 19 \end{bmatrix}$$

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PRACTICE Find the sum.

1. $$\begin{bmatrix} 3 & -9 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -8 & -4 \\ 3 & 10 \end{bmatrix}$$

2. $[5 \quad 8 \quad -4] + [-1 \quad 12 \quad 5]$

3. $$\begin{bmatrix} 3 & -9 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -8 & -4 \\ 3 & 10 \end{bmatrix}$$

4. $$4\begin{bmatrix} 2 & 7 \\ -3 & 6 \end{bmatrix} + 5\begin{bmatrix} -6 & -4 \\ 3 & 0 \end{bmatrix}$$

5. $$5\begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix} + 6\begin{bmatrix} -4 \\ 3 \\ 5 \end{bmatrix} - 2\begin{bmatrix} -3 \\ 8 \\ -4 \end{bmatrix}$$

6. $$2\begin{bmatrix} -2 & 4 \\ 1 & -1 \\ 3 & 0 \end{bmatrix} - 3\begin{bmatrix} 5 & 3 \\ -3 & 2 \\ 8 & -9 \end{bmatrix} + \begin{bmatrix} 0 & -5 \\ 9 & -3 \\ -2 & 7 \end{bmatrix}$$

7. Solve for the variables.

$$\begin{bmatrix} 4 & 2 \\ -1 & 6 \end{bmatrix} - \begin{bmatrix} x & -1 \\ 0 & z \end{bmatrix} = \begin{bmatrix} 2 & y \\ -1 & 4 \end{bmatrix}$$

DEF: A **zero matrix** is a matrix with zero for all its elements. It is also called the *additive identity matrix* because it can be added to any matrix without changing any of its elements.

Matrix *A, B*, and *C* are zero matrices because they have zero for all their elements.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

DEF: The **opposite** (or *additive inverse*) of a matrix *A* is – *A*. This means elements in the same position are the same but have opposite signs. When a matrix is added to its additive inverse the result is a zero matrix.

Matrix *D* and *E* are opposites.

$$D = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \qquad E = \begin{bmatrix} -a & -b & -c \\ -d & -e & -f \end{bmatrix}$$

$$D + E = 0$$

EXAMPLE

Find the sum.

$$\begin{bmatrix} 2 & 8 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} -2 & -8 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} (2 + -2) & (8 + -8) \\ (-3 + 3) & (0 + 0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$